in Excuſe of this Trouble, farther than that I have the Honour to be with great Reſpect,

S I R,

London, Jan. 14. 1746-7.

Your moſt obliged, and humble Servant,

Richard Brocklesby.

XIII. *A Letter from Mr.* Richard Dunthorne, *to the Rev. Mr.* Cha. Maſon, *F. R. S. and* Woodwardian *Profeſſor of* Nat. Hiſt. *at* Cambridge, *concerning the* Moon's *Motion.*

S I R, *Cambridge*, Nov. 4. 1746.

Read Feb. 5. 1746-7.

IN the Preface to my lunar Tables, I hinted, that one Uſe of publiſhing thoſe Tables would be, the aſſiſting of Perſons deſirous farther to rectify the lunar Aſtronomy, by enabling them more readily to compare the *Newtonian* Theory with Obſervations.

Since the Publiſhing thoſe Tables, I have ſpent ſome Time myſelf in that Compariſon; and here ſend you the Reſult, that you may communicate it to the *Royal Society*, if you think it deſerves to be made public.

As the Motion of every ſecondary Planet muſt partake of the Errors in the Theory of its primary, I thought proper, before I undertook the Examination of the lunar Numbers, to compare thoſe of the Sun with Obſervations. I compared ſeveral Sets of Mr.

Mr. *Flamſtead*'s Obſervations, after the Method he himſelf teaches, in *Prolegom. Hiſt. Cœleſt*, p. 133, *& ſeq.* which, for many Reaſons, I think the beſt Method hitherto uſed; and, with the Concurrence of a Gentleman well skilled in theſe Matters, determined the mean Motion of the Sun at *Greenwich*, the laſt Day of *December* at Noon, *Anno* 1700, *O. S.* ♈ 20° 43′ 40″ of its Apogee, ♋ 7° 30′ 0″, and the greateſt Equation of the Sun's Centre 1° 55′ 40″; which, I am fully perſuaded, are very near the Truth.

The Theory of the Sun being thus ſettled, I proceeded to examine the Elements of the lunar Aſtronomy. I began with Obſervations of lunar Eclipſes about the Equinoxes, when the Apogee of the Moon was in the Sun's Quadratures; becauſe at thoſe Times I could conceive the Moon's Motion affected with no Inequality, but the annual one, called by *Newton* the firſt Equation, and the elliptic one, called *Proſthaphæreſis:* From a Compariſon of ſuch Obſervations I obtained the Moon's mean Longitude, which came out 1′, at leaſt, greater than in the Tables, and very nearly as *Newton* has it in the laſt Edition of his *Principia.*

I went on to examine the Place and Motion of the Apogee, and Theory of the Increaſe and Decreaſe of the Eccentricity, as well as the greateſt and leaſt Eccentricities themſelves (from the beſt Obſervations, and beſt ſituate that I could procure) all which agreed ſo well with the Tables, about the Sun's mean Diſtances, that I dare venture to make no Alteration therein: Indeed I think the 6th Equation does not ſo well account for the Variation of the Motion of the Apogee, and Change of the Eccentricity,

tricity, according to the greater or leſſer Diſtance of the Sun from the Earth; and therefore I ſet myſelf to compute what Change this Difference of the Sun's Action upon the lunar Orbit would introduce in the Moon's Place in every Situation of the Sun and lunar Orbit; and found, after many tedious Computations, that the Sun being in Apogee, this Change, where greateſt, would amount to about $4'$, and to $4'\ 16''$, when the Sun is in Perigee. In other Diſtances of the Sun from the Earth, this greateſt Change is proportional to the Difference of the Cubes of the mean and preſent Diſtances; and in every Situation of the Moon, and of her Orbit, the preſent is to the greateſt Equation nearly as the Sine of the Exceſs of the Moon's mean Anomaly above twice the annual Argument to *Radius*. It increaſes the Moon's Longitude, when the Sun is in his $\left\{\begin{matrix}\textit{Apogeon}\\ \textit{Perigeon}\end{matrix}\right\}$ Semicircle, and that Exceſs $\left\{\begin{matrix}\text{leſs}\\ \text{greater}\end{matrix}\right\}$ than $180°$; and diminiſhes it when otherwiſe *.

In fine, I compared the Theory of the Moon, as to her Longitude, with ſeveral Obſervations, as well in the Octants and Semi-Octants, as in the Syzygies and Quadratures, and found ſuch an Agreement when the above Corrections were made, as ſeemed rather to be wiſhed than hoped for, conſidering the many Inequalities wherewith the Sun's Action diſturbs the Motion

* If this Equation be increaſed and diminiſhed in a direct *Ratio* of the Moon's horizontal Parallax, it will become more exact. And I think, if it were always diminiſhed by a fourth or perhaps a third Part, it would agree better with Obſervations.

Motion of the Moon, and the Defects to which the beſt Obſervations I have hitherto met withal are liable.

I have compared 100 obſerved Longitudes of the Moon with the Tables; *viz.* 25 Eclipſes of the Moon, all, except the firſt, taken from *Flamſtead's Hiſtoria Cœleſtis*, the *Philoſophical Tranſactions*, and the *Memoirs* of the *Royal Academy of Sciences*; the two great Eclipſes of the Sun in 1706 and 1715; 25 ſelect Places of the Moon from *Flamſtead's Hiſtoria Cœleſtis*, and 48 of thoſe Longitudes of the Moon computed from *Flamſtead's* Obſervations by Dr. *Halley* (as I ſuppoſe) printed in the firſt Edition of the *Hiſtoria Cœleſtis*. They are as follows:

25 Eclipses of the Moon, and 2 of the Sun, compared with the Tables corrected as above.

	Place of Observation.	A.D.	Apparent Time at Greenwich.				☽'s true Place observed.				M. Anom. ☉		M. Anom. ☽		Ann. Aeq.		☽'s Place computed.				Difference Observat.		
				h	′	″	s	°	′	″	s	°	s	°	s	°	s	°	′	″		′	″
a	Dantzick	1652	Septem. 7.	6	21	35	11	25	25	51	2	20	6	2	0	3	1	25	26	10	+	0	19
		1670	Sept. 18.	14	36	48	0	6	13	29	3	1	6	0	0	0½	0	6	16	37	+	3	8
	Greenwich	1678	Octob. 19.	8	21	54	1	6	47	9	4	1	7	16½	2	2	1	6	46	38	—	0	31
		1682	Febr. 11.	10	58	52	5	3	47	3	7	25	6	28½	1	14	5	3	46	14	—	0	49
	Paris	1684	Dec. 11.	10	47	6	3	1	11	20	5	24	1	8	7	16	3	1	13	[illegible]	+	1	47
	Dantzick	1685	Nov. 30.	10	35	30	2	19	40	0	5	13	11	26½	5	25	2	19	39	46	—	0	14
	Dublin	1686	Nov. 19.	11	13	11	2	8	11	57	5	2	10	12½	4	4½	2	8	11	28	—	0	29
	Greenwich	1689	Sept. 18.	14	30	37	0	6	32	8	3	1	4	24	10	8	0	6	34	16	+	2	8
		1690	March 14.	9	56	30	6	4	37	11	8	25	9	18	3	15½	6	4	37	28	+	0	17
		1696	May 6.	12	2	0	7	26	53	14	10	18	3	1½	8	28	7	26	53	40	+	0	26
	Paris	1699	March 5.	7	13	44	5	25	32	27	8	16	8	27	3	1	5	25	32	14	—	0	13
☉	New England	1703	Decem. 11.	18	29	8	3	0	49	56	5	23	11	24½	5	23	3	0	48	51	—	1	5
	Marseilles	1706	May 0.	21	13	57	1	20	46	44	10	12	6	22	7	6	1	20	46	32	—	0	12
	Greenwich		Octob. 10.	7	10	0	0	28	0	20	3	22	5	27	11	25	0	27	58	47	—	1	33
		1707	April 5.	13	39	0	6	26	18	33	9	17	11	9	5	3	6	26	19	0	+	0	26
		1708	Sept. 18.	9	10	25	0	6	39	23	2	1	2	12	8	7	0	6	39	19	—	0	4
		1712	Jan. 12.	7	34	0	4	2	57	47	6	24	1	17½	7	25½	4	2	58	37	+	0	50
	Paris	1713	Nov. 20.	15	27	30	2	9	58	21	5	3	9	21	3	17	2	9	56	56	—	1	25
☉	London	1715	April 21.	21	11	5	1	12	0	22	10	3	6	12½	6	22	1	11	59	36	—	0	46
	Paris	1717	March 15.	15	7	4	6	6	23	50	8	26	8	22½	2	27	6	6	23	31	—	0	19
		1719	August 18.	8	22	46	11	5	42	8	1	29	10	28	4	20	11	5	42	39	+	0	31
		1722	June 17.	13	46	10	9	6	47	43	11	29	5	8½	10	25	9	6	47	33	—	0	10
		1724	Octob. 20.	15	40	40	1	9	0	0	4	2	5	25½	11	22	1	8	58	50	—	1	10
		1729	Febr. 2.	8	42	55	4	25	13	39	7	16	3	25	9	14	4	25	14	40	+	1	1
			July 28.	13	0	0	10	16	15	28	1	9	8	3	2	15	10	16	16	26	+	0	58
		1731	June 8.	13	47	51	8	28	9	58	11	19	4	27	10	11	8	28	8	45	—	1	13
		1732	Nov. 20.	0	49	25	2	10	3	54	5	3	7	8	1	24	2	10	0	56	—	2	58

25 Places of the Moon, computed by myſelf from *Flamſtead*'s Obſervations, compared with the Tables.

A.D.	Apparent Time at Greenwich.	h	′	″	☽'s true Place obſerved.	s	°	′	″	M. Anom. ☉.	s	°	M. Anom. ☽.	s	°	Ann. Argt.	s	°	☽'s Place computed.	s	°	′	″	Diff. from Obſerv.	′	″
1684	March 13.	8	9	8		2	28	48	40		8	25		2	25		11	19		2	28	50	44	+	2	4
1693	March 6.	7	22	48		3	16	43	12		8	17		3	11½		11	7		3	16	44	0	+	0	48
	Octob. 11.	18	12	34		3	28	34	2		3	24		2	27		5	15		3	28	38	21	+	4	19
1694	Febr. 27.	10	29	16		4	27	27	31		8	10		3	12		9	20		4	27	26	48	−	0	43
	Auguſt 23	11	13	54		11	0	19	11		2	5		8	2		2	21		11	0	21	41	+	2	30
1694	Septem. 15.	5	34	13		8	27	1	34		2	27		6	10		3	11		8	27	1	41	+	0	7
	Septem. 21.	10	50	31		11	22	47	41		3	3		9	3½		3	17		11	22	49	16	+	1	35
	Decem. 13.	6	2	53		0	6	28	43		5	25		8	26½		6	0		0	6	29	58	+	1	15
1695	Febr. 8.	3	55	22		1	5	14	3		7	21		9	7½		7	22		1	5	12	14	−	1	49
	July 9.	5	56	14		7	2	2	42		0	20		3	14		0	2		7	2	3	4	+	0	22
	Septem. 8.	8	30	26		10	3	0	12		2	20		5	21		1	24		10	2	59	50	−	0	22
1696	January 16.	17	29	2		7	5	4	20		6	29		2	29		5	20½		7	5	6	11	+	1	51
	March 4.	9	8	13		4	12	2	24		8	16		11	8		7	3		4	12	0	59	−	1	25
1697	February 18.	6	29	47		2	20	52	29		8	2		8	19		5	10		2	20	51	7	−	1	22
	19.	7	22	57		3	4	18	16		8	3		9	2½		5	11		3	4	17	54	−	0	22
	Septem. 15.	7	54	4		10	1	7	8		2	28		3	21½		11	9		10	1	5	38	−	1	30
1698	Septem. 8.	11	2	35		11	12	15	6		2	21		3	24		9	23		11	12	15	27	+	0	21
	Novem. 27.	3	49	54		10	11	33	39		5	9		2	2½		0	3		10	11	31	13	−	2	26
1699	March 5.	12	8	23		5	28	15	59		8	16		8	29½		3	1		5	28	16	11	+	0	12
1701	Septem. 28.	6	55	23		9	27	13	17		3	10		9	9		6	8		9	27	12	49	−	0	28
1702	Octob. 16.	6	16	49		10	3	25	41		3	28		8	11½		5	13		10	3	26	27	+	0	46
1703	Septem. 13.	11	58	50		11	28	50	30		2	25		8	28		3	3		11	28	53	30	+	3	0
	Octob. 6.	6	28	39		9	27	1	44		3	18		7	4		3	23		9	27	0	38	−	1	6
1706	Octob. 10.	11	11	10		1	1	11	15		3	22		6	0		11	25		1	1	10	17	−	0	58
1714	Septem. 6.	6	34	7		9	3	44	13		2	18		3	17		0	0		9	3	45	24	+	1	11

48 Places of the Moon, computed by Dr. *Halley*, from *Flamstead*'s Observations, compared with the Tables.

A.D.	Apparent Time at Greenwich.				☽'s true Place observed.				M. Anom. ☉.		M. Anom. ☽.		Ann. Æqt		☽'s Place computed				Diff. from Observat.		
		h	′	″	s	°	′	″	°	″	s	″	s	″	s	°	′	″		′	″
1689	Novem. 16.	11	59	0	2	6	12	43	4	29	6	1	0	0	2	6	14	40	+	1	57
	Decem. 9.	6	1	0	11	28	48	54	5	21½	3	21	0	20½	11	28	48	40	—	0	14
	10.	6	46	35	0	12	46	49	5	22½	4	4	0	21½	0	12	47	14	+	0	25
	12.	8	26	33	1	12	13	8	5	24½	5	0½	0	23	1	12	10	39	—	2	29
	13.	9	24	30	1	27	38	36	5	25½	5	14	0	24	1	27	36	16	—	2	20
1690	16.	12	42	0	3	15	14	54	5	28½	6	24	0	27	3	15	15	48	+	0	54
	January 4.	3	3	46	11	11	12	59	6	17	2	24	1	14	11	11	13	8	+	0	9
	6.	4	30	15	0	8	13	31	6	19	3	20½	1	16	0	8	14	36	+	1	5
	10.	7	59	22	2	6	11	20	6	23	5	14½	1	19½	2	6	10	37	—	0	43
	12.	10	8	49	3	7	5	21	6	25	6	121	1	21	3	7	3	59	—	1	22
b	February 13.	11	14	0	3	22	36	28	6	26	6	25½	1	22	3	22	35	19	—	1	9
	2.	2	25	39	0	3	57	24	7	15½	3	5	2	10	0	3	56	9	—	1	15
	5.	4	51	10	1	16	31	33	7	18½	4	26	2	12½	1	16	34	15	+	2	42
	7.	6	48	17	2	15	58	14	7	20½	5	24	2	14½	2	15	59	13	+	0	59
	8.	7	51	54	3	0	56	20	7	21½	6	8	2	15½	3	0	56	31	+	0	11
	10.	9	56	26	4	0	55	24	7	24	7	6	2	17	+	0	55	1	—	0	23
	11.	10	52	31	4	15	42	16	7	25	7	20	2	18	4	15	41	49	—	0	27
	14.	13	19	31	5	28	12	40	7	28	9	2	2	21	5	28	11	44	—	0	56
	19.	17	3	55	8	2	23	20	8	3	11	12	2	25½	8	2	21	17	—	2	9
	21.	18	45	37	8	27	1	25	8	5	0	10	2	27½	8	26	59	6	—	2	19

1690	Febr.	22.	19	37	40	9	9	21	32	8	6	0	24	2	28	9	9	23	26	+	1	54
	March	7.	5	50	38	2	26	22	50	8	18	6	12	3	9	2	26	25	20	+	2	30
		11.	9	43	23	4	24	31	45	8	22	8	8	3	13	4	24	33	15	+	1	30
		12.	10	33	9	5	8	31	54	8	23	8	22	3	14	5	8	33	11	+	1	17
		13.	11	18	30	5	22	16	37	8	24	9	5½	3	15	5	22	17	26	+	0	49
		14.	12	2	18	6	5	44	39	8	25	9	19½	3	15½	6	5	45	18	+	0	39
		25.	20	54	54	10	24	54	20	9	6½	2	21	3	25½	10	24	53	20	−	1	0
♄	April	7.	7	48	46	4	20	8	0	9	19	8	5	4	6	4	20	9	18	+	1	18
		8.	8	36	30	5	3	59	40	9	20	8	19	4	7	5	4	0	34	+	0	54
		9.	9	22	0	5	17	34	42	9	21	9	2	4	8	5	17	35	40	+	0	58
		10.	10	5	19	6	0	54	50	9	22	9	16	4	9	6	0	55	35	+	0	45
		11.	10	47	56	6	14	0	8	9	23	9	29	4	10	6	14	2	13	+	2	5
		12.	11	30	56	6	26	55	56	9	24	10	12½	4	11	6	26	56	55	+	0	59
		13.	12	17	31	7	9	41	19	9	25	10	26	4	12	7	9	42	20	+	1	1
		14.	13	4	0	7	22	16	0	9	26	11	9½	4	12½	7	22	17	34	+	1	34
		15.	13	52	44	8	4	45	10	9	27	11	23	4	13½	8	4	44	47	−	0	23
		18.	16	26	47	9	11	43	52	10	0	1	3½	4	16	9	11	44	53	+	1	1
		20.	18	5	47	10	6	30	55	10	2	2	0	4	18	10	6	33	2	+	2	7
		22.	19	37	30	11	2	8	6	10	4	2	27	4	20	11	2	8	45	+	0	39
		24.	21	6	38	11	29	14	33	10	6	3	23½	4	21½	11	29	14	30	−	0	3
	May	14.	13	30	30	8	25	6	53	10	25½	0	7	5	8	8	25	8	48	+	1	55
♃		22.	19	41	40	0	6	50	26	11	3½	3	23	5	15	0	6	49	43	−	0	43
	June	3.	6	2	13	5	22	31	45	11	15	8	19	5	25	5	22	32	6	+	0	21
	Septem.	1.	6	48	53	9	2	27	42	2	13½	11	15	8	12	9	2	31	20	+	3	38
		8.	12	16	42	0	1	44	51	2	21	2	22	8	18	0	1	44	18	−	0	33
1691	February	23.	3	55	10	1	19	39	56	8	6	3	16	1	18	1	19	40	45	+	0	49
	March	11.	18	27	51	9	7	30	5	8	22½	10	23½	2	2	9	7	30	23	+	0	18
	Septem.	22.	8	51	3	10	19	26	24	3	4	11	16	7	19	10	19	28	32	+	2	8

a, The Time of the Middle of this Eclipſe here ſet down is from the Beginning and End; but *Hevelius* ſays he could not obſerve the Beginning exactly. Several intermediate *Phaſes* compared together ſhew the Middle to have been about 4′ ſooner; to which the Moon's Place computed is 0ˢ. 6°. 14′. 3″. and Diff. + 34″.

b, *b*, *b*, The Moon's Places, obſerved on *Feb.* 2. *April* 7. and *May* 22. are computed by myſelf, from the Obſervations; there being manifeſtly Errors, either of the Computation or Preſs, in thoſe printed in the *Hiſt. Cœleſtis*.

Several obſerved Latitudes of the Moon, which I have compared with the Tables, ſhew them to be very near the Truth, both in the Motion of the Nodes, and alſo in the Quantity and Variation of the Inclination. I am,

SIR,

Your humble Servant,

Richard Dunthorne.

XIV.